Ablation Effects on RF Attenuation in the Turbulent Boundary Layer

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Introduction

FOR slender re-entry vehicles, the significant plasma generation is restricted to the thin viscous boundary layer. The attenuation of any radio frequency (RF) signal then will be a function primarily of the peak electron concentration in the plasma sheath and of the sheath thickness. For an ablating slender re-entry vehicle, the boundary-layer thickness is dependent on the ablation rate, and the ionization may be predominantly determined by the ablation products. Although the chemistry is not well defined for most ablation products, it is possible to determine the effects of metallic atoms present in the heat shield as trace contaminants. In particular it has been shown[1] that the presence of sodium in relatively small amounts can be sufficient to render the clean air ionization insignificant. It is the purpose of this note to present a method by which the effects of trace contaminants on RF attenuation can be determined. A turbulent boundary layer is considered since in the laminar regime the ablation rate (and hence attenuation) is significantly lower.

Basic Equations and Solution

The boundary-layer equations for two-dimensional turbulent flow of a nonreacting two-component gas with zero pressure gradient are well known.[2] If it is assumed that $Pr = Pt_t = 1$ and that $Le = Le_t = 1$, the energy equation is satisfied by Crocco's relation. Thus

$$I = h_w + (u/u_e)(I_e - h_w) \tag{1}$$

where the notation is that of Dorrance.[2]

Similarly if

$$\rho D_{12}/\mu = 1 = \rho D_T/\epsilon_M \tag{2}$$

the mass fraction c is seen to be linearly related to the velocity, so that a solution to the diffusion equation is

$$c/c_w = 1 - (u/u_e) \tag{3}$$

where c represents the local contaminant unionized atom mass fraction, which will be used to calculate the local ion mass fraction.

The velocity profile in hypersonic turbulent flow is well described by a power law[3]:

$$u/u_e = (y/\delta)^{1/n} \tag{4}$$

where, in this study, n was taken as 7. Thus, as a consequence of the assumptions made, it is only necessary to describe δ to have a description of velocity, enthalpy, and mass fraction of the unionized contaminant C.

Walker and Schumann[4] provide graphical descriptions of the effect of ablation on turbulent boundary-layer thickness. Their data were used to determine δ in this study.

Ionization

The solutions of the previous section provide velocity and enthalpy distributions across the boundary layer, as well as the distribution of unionized contaminant. If equilibrium is now assumed, the degree of ionization of the contaminant can be calculated. The equilibrium constant K_p is provided

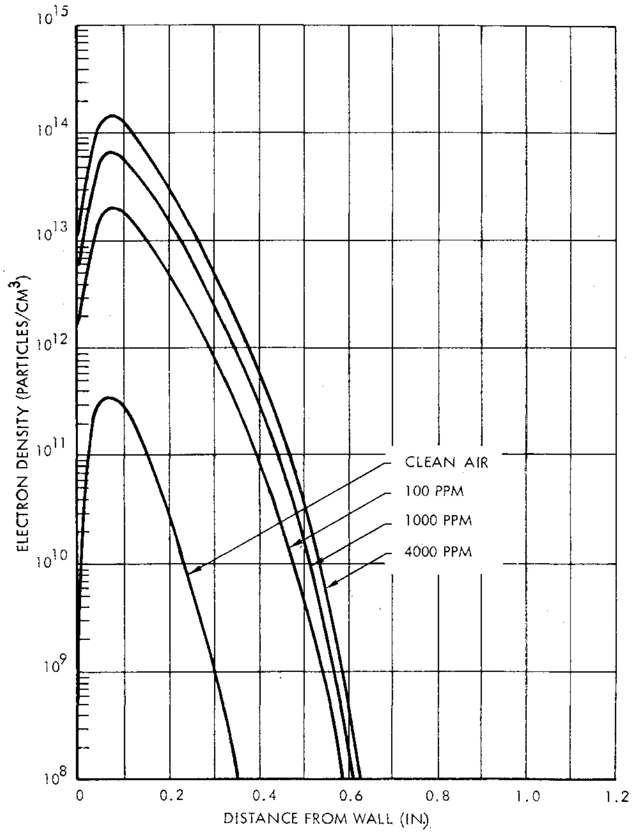

Fig. 1 Effects of sodium contamination on turbulent electron density profile.

by the equation of Saha:

$$\log K_p = \frac{-5050\,V}{T} + \frac{5}{2}\log T - 6.5 + \log \frac{g_{A^+}\cdot g_{e^-}}{g_A} \tag{5}$$

where V = ionization potential (ev), T = temperature (°K), and g = statistical weight.

If the mole fraction of contaminant is much smaller than the mole fraction of air, it can be shown that the degree of contaminant ionization is given by

$$\alpha = \{-K_p + [K_p{}^2 + 4K_p(P + K_p)X]^{1/2}\}/2(P + K_p)X \tag{6}$$

where X = the mole fraction of unionized contaminant, and P = pressure (atm). The local electron density (electrons/cm^3) due to contaminant ionization is then given by

$$N_{e^-}(y) = 0.965(10)^{22}[\rho(y)\alpha(y)C(y)/M_c] \tag{7}$$

and M_c = molecular weight of contaminant, and ρ = mixture density (lb/ft^3).

The equilibrium clean air ionization can be determined from the known local pressure and temperature and the results of Lin.[5] In this manner, the electron densities were determined over a range of altitudes and levels of contamination. Figure 1 describes such electron density profiles for a typical altitude and for several degrees of contamination.

RF Attenuation

An RF attenuation calculation was based on the one-dimensional simplified equation developed by Gold.[6] The governing equation is

$$F'' + n_0{}^2[1 - m_0X(Z)]F = 0 \tag{8}$$

where F = electric field intensity, $n_0 = 2\pi/\lambda_0$ = free space wave propagation constant, $m_0 = 1/1 - i(\omega_0/\omega)$, X =

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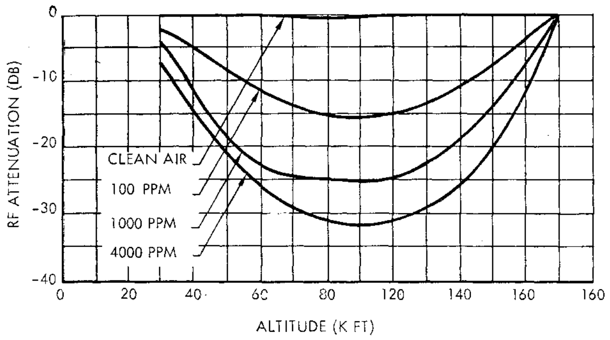

Fig. 2 RF attenuation for *C* band.

$(\omega_p/\omega)^2$. Let $X(Z)$ be a Dirac delta function

$$X(Z) = (\bar{X}L)\delta(Z) \tag{9}$$

$$\bar{X}L = \int_0^L X(Z)dZ \tag{10}$$

where $\bar{X}$ = mean electron density ratio.

Substituting Eqs. (9) and (10) into Eq. (8) and applying the boundary conditions, one obtains the simplified RF attenuation equation as follows:

$$T = 1 - R = \frac{1}{1 - i(N_0L/2)m_0X} = \frac{[1 + (\omega_c/\omega)A] + iA}{[1 + (\omega_c/\omega)A]^2 + A^2} \tag{11}$$

where

$$A = \frac{\pi}{\lambda}\frac{\left\{\int_0^L \left(\frac{\omega_p}{\omega}\right)^2 dz\right.}{\left.\left[1 + \left(\frac{\omega_c}{\omega}\right)^2\right]\right\}} \tag{12}$$

The attenuation equation in db becomes

$$\text{db} = 10 \log_{10} |T|^2 =$$

$$10 \log_{10}\left(\left\{\frac{[(1 + (\omega_c/\omega)A]}{[1 + (\omega_c/\omega)A]^2 + A^2}\right\}^2 + \left\{\frac{A}{[1 + (\omega_c/\omega)A]^2 + A^2}\right\}^2\right) \tag{13}$$

This equation was programed by Vicenti and Fletcher of Aerospace Corporation. The electron density profiles calculated by the method presented in the previous section are used as inputs for this program. The results of attenuation of an electromagnetic wave passing through the plasma sheath of a slender cone case are discussed in the next section.

Results and Conclusion

The attenuation of an RF signal generated by an 11°-cone flying a typical re-entry trajectory was studied. Figure 1 describes the electron density profiles at an altitude of 60,000 ft for various degrees of sodium contamination. The cone was considered at zero angle of attack with a velocity of 19,000 fps. Figures 2 and 3 describe the attenuation as a function of altitude for the various contaminant levels. For altitudes greater than 100,000 ft, a laminar boundary-layer analysis was used.[1] The pronounced effect of contaminants is obvious. Not only do they raise the peak electron concentration (Fig. 1), but the blowing significantly thickens the boundary layer, adding to the attenuation. For an ablating heat shield, the sodium contribution to RF attenuation completely predominates over the clean air contribution and is predictable by the previous simplified analysis.

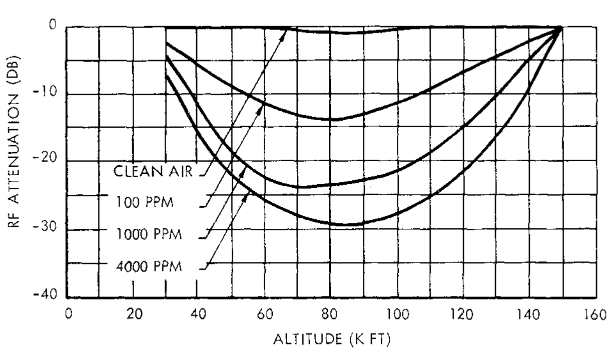

Fig. 3 RF attenuation for *X* band.

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Creep under Random Loading

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Introduction

THE analysis of creep deformation under random loading should be based on a stress-strain relation that is valid for arbitrarily variable stress conditions. For linear viscoelastic materials, in the case of uniaxial stress, such a relation can be written in the form[1,2]

$$\epsilon(t) = \frac{1}{E}\left[\sigma(t) + \int_0^t \varphi(t - \theta)\sigma(\theta)d\theta\right] \tag{1}$$

where $\sigma(t)$ is the stress, $\epsilon(t)$ is the corresponding strain, and E and $\varphi(t)$ are a material constant and a material function, respectively. The constant E determines the instantaneous part of strain, which is proportional to the stress. The function $\varphi(t)$ is related to the rate of creep under constant stress, and the integral in Eq. (1) represents the time-dependent part of strain.

For metals at elevated temperatures, the following nonlinear stress-strain relation has been suggested and experimentally verified[3–5]:

$$\epsilon(t) = \frac{\sigma(t)}{E_M} + \frac{1}{\eta_K}\int_0^t e^{-(t-\theta)E_K/\eta_K} + \int_0^t \frac{\sigma(\theta)}{\eta_M(\theta)}\,d\theta \tag{2}$$

where E_M, E_K, and η_K are material constants, whereas $\eta_M(\theta)$ is a function of stress which may be assumed in the form

$$\eta_M(\sigma) = [B\,|\sigma|^{n-1}]^{-1} \tag{3}$$

with B and n being material constants. The first term on the right-hand side of Eq. (1) is the elastic part of strain, the

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